Problems marked with ** are only relevant for MATH 2551.

PROBLEMS

1. Find $T$, $N$, and curvature for $r(t) = (3 \sin t)\hat{i} + (3 \cos t)\hat{j} + 4t\hat{k}$.

2. **Write acceleration in terms of tangential and normal components for $r(t) = (t + 1)\hat{i} + 2t\hat{j} + t^2\hat{k}$, $t = 1$.

3. Find the limit: $\lim_{(x,y) \to (0,0)} \frac{3x^2 - y^2 + 5}{x^2 + y^2 + 2}$.

4. Find all second-order partial derivatives for $w = x \sin (x^2 y)$.

5. Evaluate $\frac{dw}{dt}$ for $w = 2ye^x - \ln z$, $x = \ln (t^2 + 1)$, $y = \arctan t$, $z = e^t$, $t = 1$.

6. Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ for $z^3 - xy + yz + y^3 - 2 = 0$, $(1, 1, 1)$.

7. Find the derivative of the function at the point in the direction of $\vec{v}$ for $f(x, y) = 2xy - 3y^2$, $P = (5, 5)$, $\vec{v} = 4\hat{i} + 3\hat{j}$.

8. Find the tangent plane and normal line for $2z - x^2 = 0$, $P_0(2, 0, 2)$.

9. Find all local maxima, minima, and saddle points for $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$.

10. Find absolute maxima and minima for $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0$, $y = 2$, $y = 2x$ in the first quadrant.

11. Find the maximum value of $f(x, y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$.

12. Write an iterated integral using (a) vertical cross-sections and (b) horizontal cross-sections for $y = \sqrt{x}$, $y = 0$, $x = 9$.

13. Sketch the region and evaluate $\int_0^1 \int_0^{y^2} 3y^3 e^{-xy} dxdy$.

14. Change to polar and evaluate $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} dydx$.

15. For the region below $z = 4 - xy$ and in $0 \leq x \leq 2$, $0 \leq y \leq 1$, integrate the function $3 - 4x$.

16. Use $x = \frac{u}{v}$, $y = uv$ in $\mathbb{R}$ (first quadrant) bounded by $xy = 1$, $xy = 9$, $y = x$, $y = 4x$ for $\int \int (\sqrt{\frac{y}{x}} + \sqrt{\frac{x}{y}}) dxdy$.

17. **Evaluate the line integral $\int (xy + y + z)ds$ along the curve $r(t) = 2t\hat{i} + t\hat{j} + (2 - 2t)\hat{k}$, $0 \leq t \leq 1$.

18. **Find the flow of the field $F = -4xy\hat{i} + y8\hat{j} + 2\hat{k}$ along $r(t) = t\hat{i} + t^2\hat{j} + \hat{k}$, $0 \leq t \leq 2$.

19. **Find the circulation and flux of the field $F = x\hat{i} + y\hat{j}$ around $r(t) = \cos t\hat{i} + \sin t\hat{j}$, $0 \leq t \leq 2\pi$.

20. **Find the potential function for $F = e^{y+2z}(\hat{i} + x\hat{j} + 2x\hat{k})$. 
21. **Use Green’s Theorem to find counterclockwise circulation and outward flux for field \( F = (y^2 - x^2) \mathbf{i} + (x^2 + y^2) \mathbf{j} \) and curve \( C: y = 0, x = 3, y = x \).

22. **Use a parametrization to express the area of the surface as a double integral: the portion of the cone \( z = 2\sqrt{x^2 + y^2} \) between the planes \( z = 2 \) and \( z = 6 \).

23. Evaluate \( \int \int_S 2y \, dV \) over \( S: y^2 + z^2 = 4 \) between \( x = 0, x = 3 - z \).

24. At time \( t = 0 \), a particle is located at the point \((0, 1, 2)\). At this time it is traveling towards the point \((3, 2, 3)\), has speed 3 at \((0, 1, 2)\), and has constant acceleration \(3\mathbf{i} - \mathbf{j} + \mathbf{k}\). Find an equation for the position vector \( \mathbf{r}(t) \) at time \( t \).

25. Let \( P(A_0, B_0, C_0), Q(A_1, B_1, C_1) \) be two distinct points in 3-dimensional space.
   (a) Write down a vector parameterization for the line \( PQ \).
   (b) Show that the curvature \( \kappa \) of \( PQ \) is 0.
   (c) **Show that the torsion \( \tau \) of \( PQ \) is 0.

26. (a) At what points \((x, y)\) in the plane is the function \( f(x, y) = \frac{y}{1 + \cos x} \) continuous?
   (b) Find the following limit by first rewriting the fraction \( \lim_{(x, y) \to (1, -1)} \frac{x^3 + y^3}{x + y} \).

27. Let \( C \) be the smooth curve given by the intersection of the two surfaces \( xyz = 1 \) and \( x^2 + 2y^2 + 3z^2 = 6 \).
   (a) Write down an equation that describes \( C \) implicitly.
   (b) Write down a parametric equation for the line tangent to \( C \) at the point \((1, 1, 1)\).

28. Let \( D \) be the cylinder bounded below by \( z = -1 \), bounded on the sides by \( x^2 + y^2 = 1 \), and bounded above by \( z = 1 \). (Do not evaluate the integrals.)
   (a) Express the volume of \( D \) as an iterated triple integral in cylindrical coordinates.
   (b) Express the volume of \( D \) as an iterated triple integral in rectangular coordinates.
   (c) Express the volume of \( D \) as an iterated triple integral in spherical coordinates.

29. Consider the function \( f(x, y) = \frac{\ln(x^2 + y^2)}{\sqrt{x^2 + y^2}} \).
   (a) Describe the region \( R \) given by \( 1 \leq x^2 + y^2 \leq e \) in polar coordinates.
   (b) Write down the integral of \( f(x, y) \) over \( R \) in polar coordinates.
   (c) Evaluate the integral you found in part (b).

30. Let \( D \) be the region in \( xyz \)-space defined by the inequalities \( 1 \leq x \leq 2, 0 \leq xy \leq 2, 0 \leq z \leq 1 \). Consider the coordinate transformation of \( D \) to the \( uvw \)-plane given by \( u = x, v = xy, w = 3z \).
   (a) Sketch the preimage \( G \) of \( D \) under the coordinate transformation in the \( uvw \)-plane and label the bounding curves.
   (b) Write down the Jacobian associated to this coordinate transformation.
   (c) Evaluate the integral \( \iiint_D (x^2 y + 3xyz) \, dx \, dy \, dz \).

31. A flat circular plate has the shape of the region \( x^2 + y^2 \leq 1 \). Points on the plate have temperature \( T(x, y) = x^2 + 2y^2 - x \). Find the temperatures of the hottest and coldest points of the plate.

32. Find the points on the surface \( xyz = 1 \) closest to the origin.

33. Find the volume of the wedge cut from the cylinder \( x^2 + y^2 \leq 1 \) by the planes \( z = -y \) and \( z = 0 \).
1. \( T = \frac{3\cos t}{5} \, \vec{i} - \frac{3\sin t}{5} \, \vec{j} + \frac{4}{5} \, \vec{k} \), \( N = (-\sin t) \, \vec{i} - (\cos t) \, \vec{j}, \) \( \kappa = \frac{3}{25} \)

2. \( a(1) = \frac{4}{3} T + \frac{2\sqrt{5}}{3} N \)

3. \( \frac{5}{2} \)

4. \( w_{xx} = 6xy\cos(x^2y) - 4x^3y^2\sin(x^2y), \ w_{yy} = -x^5\sin(x^2y), \ w_{xy} = 3x^2\cos(x^2y) - 2x^4y\sin(x^2y) \)

5. \( \frac{dw}{dt} = 4t \arctan t + 1 = \pi + 1 \)

6. \( \frac{1}{4}, -\frac{3}{4} \)

7. \(-4\)

8. \(2x - z - 2 = 0, \ r(t) = (2, 0, 2) + t(-4, 0, 2)\)

9. \( f(-3, 3) = -5 \) minimum

10. \( (0, 0) = 1, \) abs max, \( (1, 2) = -5, \) abs min

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12. \(0 \leq x \leq 9, \ 0 \leq y\sqrt{x}, \ 0 \leq y \leq 3, \ y^2 \leq x \leq 9\)

13. \( c - 2\)

14. \( \frac{5}{2} \)

15. \( \int_0^2 \int_0^1 \int_0^{1-x} (3 - 4x)dz \, dy \, dx = -\frac{17}{3} \)

16. \( \int_1^2 \int_1^3 \frac{(u+v)2u}{v} \, du \, dv = 8 + \frac{52}{3} \ln 2 \)

17. \( \frac{13}{2} \)

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19. Circulation = 0; Flux = \(2\pi\)

20. \( f(x, y, z) = xe^{y+2z} + C\)

21. Flux = \(-9\), Circulation = 9

22. \( \int_0^{2\pi} \int_1^3 r\sqrt{5} \, dr \, d\theta \)

23. \( r(u, v) = r(x, \theta) = (x, 2\sin \theta, 2\cos \theta) \)
\[ \int \int f(r(u, v))|r_u \times r_v| \, dA = \int_0^{2\pi} \int_0^{3-2\cos \theta} 2(2\sin \theta)(2) \, dx \, d\theta = 0 \]

24. \( \vec{r}(t) = \left( \frac{1}{2}t^2 + \frac{9}{\sqrt{11}} t \right) \, \vec{i} + \left( -\frac{1}{2}t^2 + \frac{3}{\sqrt{11}} t + 1 \right) \, \vec{j} + \left( \frac{1}{2}t^2 + \frac{3}{\sqrt{11}} t + 2 \right) \, \vec{k} \)

25. (a) \( L(t) = (x(t) = A_0 + (A_1 - A_0)t, y(t) = B_0 + (B_1 - B_0)t, z(t) = C_0 + (C_1 - C_0)t) \)

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26. (a) \( f \) is continuous on \( \{(x, y) | x \neq (2k + 1)\pi; k = \cdots, -1, 0, 1, \cdots \} \). (b) 1

27. (a) \( x^2 + 2y^2 + 3z^2 = 6xyz \). (b) \( L(t) = (1 + 2t, 1 - 4t, 1 + 2t) \)

28. (a) \( \int_{-1}^{1} \int_{0}^{2\pi} \int_{0}^{1} r \, dr \, d\theta \, dz \). (b) \( \int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{-1}^{1} dz \, dx \, dy \)
   (c) \( \int_{0}^{2\pi} \int_{0}^{\pi/4} \int_{0}^{1/\sin \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta + 2 \int_{0}^{2\pi} \int_{0}^{\pi/4} \int_{0}^{1/\cos \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \)

29. (a) \( 1 \leq r^2 \leq e \). (b) \( \int_{1}^{2\pi} \int_{1}^{\sqrt{2} \ln (r)} 2 \ln (r) \, dr \, d\theta \). (c) \(-2\pi \sqrt{e} + 4\pi \)

30. (b) \( \frac{1}{10} \). (c) \( 2 + 3 \ln(2) \)

31. Hottest: \( \frac{9}{4} \) at points \((-\frac{1}{2}, \frac{\sqrt{3}}{2}) \), \((-\frac{1}{2}, -\frac{\sqrt{3}}{2}) \). Coldest: \(-\frac{1}{4} \) at point \((\frac{1}{2}, 0) \).

32. Points \((1, 1, 1), (-1, -1, -1), (1, -1, -1), \) and \((-1, 1, -1)\) are all at distance 3 from the origin.

33. \( \frac{2}{3} \)